

St Louis Community College Department of Mathematics &
St. Louis Community College Foundation

Present the 50th Annual

EXCELLENCE IN MATHEMATICS COMPETITION

St. Louis Area High School Mathematics Contest
November 5, 2022

With Special Thanks to



AND



Prepared with help from the
American Mathematical Association of Two Year Colleges
and the Mathematics Department of
Rose-Hulman Institute, Terre Haute, Indiana

Instructions:

- You have 63 minutes to complete the items in this packet.
- Record all your responses on the answer sheet in spaces labeled # 1 through 20.
- Use the last page of this packet for scratch paper.
- No electronic or mechanical calculators are allowed.
- Each page has five items and every participant answers the same set of items.
- Each correct response is worth 5 points, no response earns 0 points, and every incorrect response earns -1 point.
- Do not mark your answer sheet during the first 60 minutes. Instead, write your responses down in this packet. You will receive clear instructions to mark your answer sheet with your final responses during the final three minutes. This will help you minimize erasures, which can affect your score negatively.
- Write all information in clear, block, capital letters

Scoring Note: In the event of a tie score, item #20 is used as a tie-breaker. If ties still remain, item #19 will be used a tie-breaker, etc. A tie is resolved/won only with a correct response. Team ties are resolved based on the highest individual scorer for each team.

ITEM 1. A collection of coins is made up of an equal number of pennies, nickels, dimes, and quarters. What is the largest possible value of the collection which is less than \$2.00?

- A) \$1.64 B) \$1.78 C) \$1.86 D) \$1.89 E) none of these

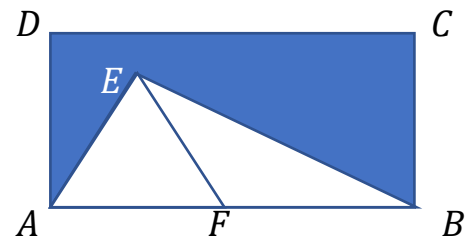
ITEM 2. On a certain trip, Maria travels $60 \frac{ft}{s}$ for half the distance and $50 \frac{ft}{s}$ for the rest of the trip. What is her average speed in $\frac{ft}{s}$?

- A) $54 \frac{6}{11}$ B) $54 \frac{8}{11}$ C) $54 \frac{10}{11}$ D) $55 \frac{6}{11}$ E) none of these

ITEM 3. The sum of all integers between 0 and 101 that are perfect cubes is

- A) 68 B) 87 C) 100 D) 110 E) none of these

ITEM 4. In the figure shown, suppose that $ABCD$ is a $4 ft$ by $8 ft$ sheet of plywood. F is the midpoint of AB , which has length $8 ft$, and AEF is an equilateral triangle. The triangular piece AEB is kept and the shaded portion is wasted. How many square feet of plywood are wasted?



- A) $32 - 16\sqrt{3}$ B) $32 - 8\sqrt{3}$ C) $32 - 6\sqrt{3}$ D) $32 - 4\sqrt{3}$ E) none of these

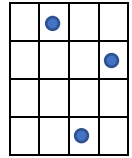
ITEM 5. $x \blacksquare y = xy + 2x$, then find the value of a so that $2 \blacksquare (a \blacksquare 1) = 22$

- A) 1 B) 2 C) 3 D) 10 E) none of these

ITEM 6. A door is four feet wide and seven feet high. If the door is standing open at a 90° angle with the door frame, what is the greatest distance in feet from the outer top corner of the door to a point on the door frame?

- A) 7 B) 8 C) 8.5 D) 9 E) none of these

ITEM 7. In the 4×4 grid shown three coins are randomly placed in different squares. The probability that no two coins lie in the same row or column is



- A) $\frac{9}{64}$ B) $\frac{6}{35}$ C) $\frac{7}{40}$ D) $\frac{9}{65}$ E) none of these

ITEM 8. The sides of a square are increased by two, making the area three times larger. The length of one side of the original square was

- A) $1 + \sqrt{3}$ B) $\sqrt{3} - 1$ C) $2\sqrt{3}$ D) $\frac{\sqrt{3}}{2}$ E) none of these

ITEM 9. Let n be an integer greater than or equal to 3. When simplified, the product $\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{5}\right) \dots \left(1 - \frac{1}{n}\right)$ becomes

- A) $\frac{n-2}{n}$ B) $\frac{1}{n}$ C) $\frac{2}{n}$ D) $\frac{n}{n-2}$ E) none of these

ITEM 10. Three people (X, Y, Z) are in a room with you. One is a knight (knights always tell the truth), one is a knave (knaves always lie), and the other is a spy (spies may either lie or tell the truth). X says, "Y is the spy." Y says "Z is the spy." Z says "Y is the spy." Which of the following correctly identifies all three people?

- A) X is the knight Y is the spy Z is the knave
 B) X is the spy Y is the knave Z is the knight
 C) X is the knight Y is the knave Z is the spy
 D) X is the knave Y is the knight Z is the spy

E) none of these

- ITEM 11.** Two adjacent faces of a rectangular box have areas 36 and 63 square units. If all three dimensions are positive integers, find the ratio of the largest possible volume of the box to the smallest possible volume
- A) 1 B) 3 C) 9 D) 12 E) none of these
- ITEM 12.** At 1:10 pm, the acute angle formed by the minute hand and the hour hand of an analog clock is:
- A) 22.5° B) 25° C) 27.5° D) 30° E) none of these
- ITEM 13.** If a convex polygon has n sides and exactly n diagonals, then n is equal to
- A) 4 B) 6 C) 8 D) 10 E) none of these
- ITEM 14.** Teams A and B play a series of games; whoever wins two games first wins the series. If Team A has a 70% chance of winning any single game, what is the probability that Team A wins the series?
- A) 0.500 B) 0.625 C) 0.700 D) 0.784 E) none of these
- ITEM 15.** If positive integers a and b are solutions to $\frac{1}{a} = \frac{1}{5} - \frac{1}{b}$, how many distinct values of a are possible?
- A) 0 B) 1 C) 2 D) 3 E) none of these

ITEM 16. A rectangular tabletop is made from 1000 square tiles of the same size with no space between them. It has dimensions 28" by 17.5". What percentage (rounded to the nearest tenth of a percent) of the total area of the tabletop consists of edge tiles (that is, tiles that have one or two edges along the outside edge)?

- A) 11.5 B) 12.2 C) 12.6 D) 13 E) none of these

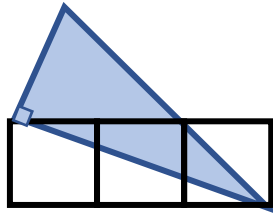
ITEM 17. What is the hundreds digit in the product $5^{94} \cdot 98,777,782,163$?

- A) 0 B) 2 C) 4 D) 8 E) none of these

ITEM 18. If x and y are nonzero numbers such that $x = 1 + \frac{1}{y}$, and $y = 1 + \frac{2}{x}$, then y is also equal to

- A) $x - 1$ B) $2x$ C) $1 + x$ D) $-x$ E) none of these

ITEM 19. The figure below shows three squares, each with area equal to four square units. What is the area of the shaded right triangle in square units?



- A) 10 B) 12 C) 14 D) 16 E) none of these

ITEM 20. Some children are playing a game that uses a regular octagon $ABCDEFGH$. There are pennies on some of the sides: 1 on \overline{AB} , \overline{BC} , and \overline{EF} ; 3 on \overline{CD} ; 2 on \overline{DE} ; and none on \overline{FG} , \overline{GH} , and \overline{HA} . Each child, in turn, may add a penny to each of two adjacent sides (for example, a child may add a penny to \overline{AB} and a penny to \overline{BC}), but no other changes are permitted. Their goal is to reach a state where all sides have the same number of pennies. If S is the smallest number of turns needed, which inequality does S satisfy?

- A) $S \leq 8$ B) $8 \leq S \leq 15$ C) $15 < S \leq 25$ D) $S < 25$

E) none of these

