St. Louis Community College Department of Mathematics &
St. Louis Community College Foundation

Present the 52nd Annual

EXCELLENCE IN MATHEMATICS COMPETITION

St. Louis Area High School Mathematics Contest
November 2, 2024

With Special Thanks to

WOLFRAM   AND   TEXAS INSTRUMENTS

Instructions:
- You have <u>63 minutes</u> to complete the items in this packet.
- Record all your responses on the answer sheet in spaces labeled # 1 through 20.
- Use the last page of this packet for scratch paper.
- No electronic or mechanical calculators are allowed.
- Each page has five items and every participant answers the same set of items.
- Each correct response is worth 5 points, no response earns 0 points, and every incorrect response earns -1 point.
- Do not mark your answer sheet during the first 60 minutes. Instead, write your responses down in this packet. You will receive clear instructions to mark your answer sheet with your final responses during the final three minutes. This will help you minimize erasures, which can affect your score negatively.

Scoring Note: In the event of a tie score, item #20 is used as a tie-breaker. If ties still remain, item #19 will be used a tie-breaker, etc. A tie is resolved/won only with a correct response. Team ties are resolved based on the highest individual scorer for each team.

ITEM 1. In a group of 100, each a Morp or a Lerp, every Morp has 2 Lerp friends and no Lerp has more than 1 Morp friend. If exactly 16 Lerp have no Morp friends, then how many Lerps are in the group?

A) 24   B) 48   C) 65   D) 72   E) none of these

ITEM 2. Suppose you own a convenience store. You purchase hotdogs and buns in bulk. Cases of hot dogs contain 156 hot dogs. Cases of buns contain 182 buns. You need to have the same number of hot dogs and buns. Find the FEWEST amount of cases of buns you can buy, so that you will be able to purchase the same number of hot dogs and buns.

A) 7   B) 13   C) 26   D) 42   E) none of these

ITEM 3. A rectangle was altered by increasing its length by 10 percent and decreasing its width by m percent. If these alterations decreased the area of the rectangle by 12 percent, what is the value of m?

A) 5   B) 10   C) 20   D) 25   E) none of these

ITEM 4. On an exam 20% of the students scored 70%, 40% scored 80%, 30% scored 90% and the rest scored 100%. What is mean score of the students in the class?

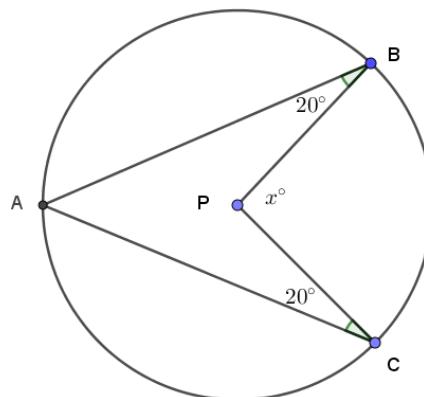A) 25%   B) 68%   C) 83%   D) 95%   E) none of these

ITEM 5. A semicircle is tangent to both legs of a right triangle and has its center on the hypotenuse. The hypotenuse is partitioned into 4 segments, with lengths 3, 12, 12, and x, as shown. What is the value of x?
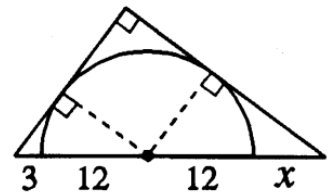
A) 1   B) 2   C) 4   D) 8   E) none of these

ITEM 6. If P is the center of the circle, find x.

A) 40 degrees
B) 80 degrees
C) 60 degrees
D) 90 degrees
E) none of these

ITEM 7. If $3x - y = 12$, then find the value of $\sqrt{\dfrac{8^x}{2^y}}$

A) 64   B) 16   C) 4   D) 32   E) none of these

ITEM 8. Sally rows 12 miles per hour in still water. In a river that flows at a constant rate, she rows upstream to reach her destination in 2 hours. Show rows back the same distance in 1 hour. At what rate was the river flowing in miles per hour?

A) 3   B) 4   C) 5   D) 6   E) none of these

ITEM 9. What is the remainder if 3^{21} is divided by 5?

A) 1   B) 2   C) 3   D) 4   E) none of these

ITEM 10. In a class, 21 students passed in Math, 26 passed in Physics, 29 passed in Chemistry, 14 students passed in both Math and Physics, 12 students passed in both Math and Chemistry, 14 students passed in both Physics and Chemistry, and 8 passed in all three subjects. Then how many passed in Chemistry only?

A) 10   B) 11   C) 12   D) 15   E) none of these

ITEM 11. The value of $\left[i^{18} + \left(\frac{1}{i} \right)^{25} \right]^3$ is:

A) 2-2i   B) 2+2i   C) 2i   D) 2   E) none of these

ITEM 12. The number of words with (or) without meaning using all the letters of UNITEDSTATES such that all the vowels come together is:

A) 201,600   B) 216,000   C) 206,100   D) 200,160   E) none of these

ITEM 13. A geometric sequence is one in which the terms of the sequence are multiplied by a common value. For example, the sequence 2,6,18,54,162, … is a geometric sequence where each term is multiplied by three to get the subsequent term. Suppose the 1st term of a geometric sequence is a and 3rd term is b. Suppose further that in a second, different geometric sequence, the 1st term is a and 5th term is b then the 11th term of first sequence is common to which term of the second sequence?

A) 16   B) 24   C) 18   D) 21   E) none of these

ITEM 14. Suppose a virus has infected 1% of the population at this given time. A test used to detect the virus in a person is positive 86% of the time if the person has the virus (true positive) and 10% of the time if the person does not have the virus (false positive). Suppose a random person takes the test and obtains a positive test result. Find the probability the person has the virus (rounded to 3 decimal places).

A) 0.08   B) 0.086   C) 0.86   D) 0.9   E) none of these

ITEM 15. Given two congruent circles with radius r tangent to each other and to a line as shown, and a square of side length t, with one side resting on the line and touching both circles at vertices that share a side, what is the ratio of t to r?
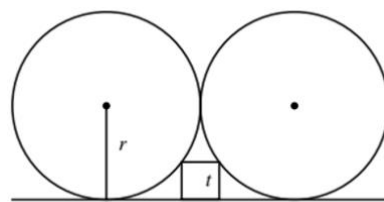
A) $\frac{1}{2}$ B) $1 - \sqrt{2}$ C) $\sqrt{3}$ D) $\frac{2}{5}$ E) none of these

ITEM 16. On a ten question multiple choice exam, each question has the same number of choices as the problem number (e.g. problem #7 has options a,b,c,d,e,f, and g). A correct answer is worth the square of the problem number points (so answering number 4 correctly is worth 16 points). An incorrect answer subtracts the problem number from your score. If you randomly guess on each problem how many points can you expect to score?

A) -4   B) 1   C) 4   D) 9   E) none of these

ITEM 17. The sequence of digits 123456789101112131415… is found by writing the counting numbers 1,2,3,… in ascending order. What would be the 2024th digit from the left?

B) 0   B) 1   C) 2   D) 3   E) none of these

ITEM 18. The sum of the fourth powers of three positive integers is 1633. Find the sum of the three integers.

A) 8   B) 12   C) 14   D) 16   E) none of these

ITEM 19. At which time between 7:30 am and 8:00 am will the angle between hour hand and minute hand be exactly 90 degrees? Round to the nearest second.

A) 7:54:33   B) 7:50:23   C) 7:51:13   D) 7:55:33   E) none of these

ITEM 20. If the centroid of a triangle which is formed by the points (a, b), (b, c), and (a, c) is at the origin, then the value of $a^3 + b^3 + c^3$ is:

A) abc B) $3abc$ C) $a + b + c$ D) 0   E) none of these